REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
06-12-2012	Conference Proceeding	-

4. TITLE AND SUBTITLE

The complexity of the Quantum Adiabatic Algorithm

5a. CONTRACT NUMBER
W911NF-09-1-0391

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS

A.P. Young, S. Knysh, V.N. Smelyanskiy

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES

University of California - Santa Cruz
Office of Sponsored Projects
The Regents of the University of California, Santa Cruz
Santa Cruz, CA 95064 -1077

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
56290-PH-QC.6

12. DISTRIBUTION AVAILIBILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

The Quantum Adiabatic Algorithm has been proposed as a general purpose algorithm for solving hard optimization problems on a quantum computer. Early work on very small sizes indicated that the running time (complexity) only increased as a (quite small) power of the problem size N. We report results of Quantum Monte Carlo simulations, using parallel tempering, with which we determine the minimum energy gap (and hence get information the complexity) for much bigger sizes than was possible before. The aim is to see if there is a “crossover” to

15. SUBJECT TERMS

Quantum Adiabatic Algorithm, Quantum Monte Carlo

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Peter Young
UU	UU	UU	UU		19b. TELEPHONE NUMBER: 831-459-4151

Report Title

The complexity of the Quantum Adiabatic Algorithm

ABSTRACT

The Quantum Adiabatic Algorithm has been proposed as a general purpose algorithm for solving hard optimization problems on a quantum computer. Early work on very small sizes indicated that the running time (complexity) only increased as a (quite small) power of the problem size N. We report results of Quantum Monte Carlo simulations, using parallel tempering, with which we determine the minimum energy gap (and hence get information the complexity) for much bigger sizes than was possible before. The aim is to see if there is a "crossover" to exponential complexity at large N. We present data for the typical (median) complexity as a function of N, which indicate a crossover to a first order transition at large sizes. This implies that the complexity is exponential at large N, at least for the problem studied.

Conference Name: Conference on Computational Physics Kaohsiung, Taiwan,

Conference Date:

Computer Physics Communications 182 (2011) 27–28

Contents lists available at ScienceDirect

Computer Physics Communications

www.elsevier.com/locate/cpc

The complexity of the Quantum Adiabatic Algorithm

A.P. Young [a,*], S. Knysh [b], V.N. Smelyanskiy [c]

[a] Physics Dept., University of California Santa Cruz, Santa Cruz, CA 95064, USA
[b] ELORET Corporation, NASA Ames Research Center, MS 229, Moffett Field, CA 94035-1000, USA
[c] NASA Ames Research Center, MS 269-3, Moffett Field, CA 94035-1000, USA

ARTICLE INFO

Article history:
Received 13 February 2010
Accepted 2 June 2010
Available online 8 June 2010

Keywords:
Complexity
Quantum
Algorithm

ABSTRACT

The Quantum Adiabatic Algorithm has been proposed as a general purpose algorithm for solving hard optimization problems on a quantum computer. Early work on very small sizes indicated that the running time (complexity) only increased as a (quite small) power of the problem size N. We report results of Quantum Monte Carlo simulations, using parallel tempering, with which we determine the minimum energy gap (and hence get information the complexity) for much bigger sizes than was possible before. The aim is to see if there is a "crossover" to exponential complexity at large N. We present data for the typical (median) complexity as a function of N, which indicate a crossover to a first order transition at large sizes. This implies that the complexity is exponential at large N, at least for the problem studied.

Interest in quantum computers has been high since Shor [1] introduced an algorithm for factoring large integers which is much more efficient than any known algorithm on a classical computer. The issue we address here is whether quantum computers could, in addition, solve a wide range of *hard optimization problems* more efficiently than a classical computer.

Of particular interest is the set of NP-complete problems, for which there is no known classical algorithm which will solve the problem in polynomial time (i.e. a time which increases as a power of the problem size N) for the worst case instances, and, in general, for typical instances too. Classically one distinguished between problems which can be solved in polynomial time, class P, from those for which the solution can only be *verified* in polynomial time, class NP. It is widely believed, but not proved, that P is a subset of NP and that the NP-complete set is a (different) subset of NP, see the left part of Fig. 1. Integer factoring is in a set BQP, which contains those problems which can be solved in polynomial time on a quantum computer. On account of Shor's algorithm, it appears that BQP is larger than P. If BQP were actually to contain NP-complete, this would give a big additional motivation to experimentalists struggling to overcome the severe technical difficulties in building a quantum computer. Determining the efficiency of a quantum computer for solving one particular NP-complete problem is the objective of this study.

Farhi et al. [2] proposed the Quantum Adiabatic Algorithm (QAA) as an algorithm for solving optimization problems on a quantum computer. In this approach, which is related to "quantum annealing", one simulates the Hamiltonian in the quantum computer by an appropriate choice of interactions between the qubits. One adds to the "problem Hamiltonian", $\mathcal{H}_P$, whose ground state we wish to find and which is expressed in terms of Ising spin variables $\sigma_i^z \pm 1$, a non-commuting driver Hamiltonian, $\mathcal{H}_D$, the simplest example of which is just a transverse field on each site, so $\mathcal{H}_D = -\sum_i \sigma_i^x$, where σ_i^z and σ_i^x are Pauli spin matrices. At time $t = 0$ the system is started off in the ground state of $\mathcal{H}_D$, and at subsequent times, the amount of $\mathcal{H}_D$ is decreased and that of $\mathcal{H}_P$ is increased until, at the end of the run at time $\mathcal{T}$, one only has $\mathcal{H}_P$. If the process is sufficiently slow that it is adiabatic, the quantum computer ends up in the ground state of $\mathcal{H}_P$ and the problem is solved. We will denote the time $\mathcal{T}$ to obtain the correct answer with some significant probability as the "complexity" of the problem. The interest is in how the complexity varies with N; in particular is it polynomial or exponential. The bottleneck will be at a point where the gap between the ground state and the first excited state becomes very small during the time-evolution of the system. This can happen at a quantum phase transition. Early work [2] on very small sizes, $N \lesssim 20$, that $\mathcal{T} \sim N^2$, i.e. polynomial complexity.

To see if polynomial complexity is also true for $N \to \infty$ or whether there is a "crossover" to exponential complexity for large sizes, we have carried out [3,4] quantum Monte Carlo (QMC) simulations on the particular NP-complete "constraint satisfaction problem" studied by Farhi et al. [2], known as Exact Cover. We chose instances with a "unique satisfying assignment" and found [4] that the quantum phase transition turns discontinuous (first order) as the size increases. The right part of Fig. 1 shows the fraction of instances with a first order transition as a function of size. It seems likely that this fraction tends to unity for $N \to \infty$. It is expected that the minimum energy gap is exponentially small [5] at a first

* Corresponding author.
E-mail address: peter@physics.ucsc.edu (A.P. Young).

0010-4655/$ – see front matter
doi:10.1016/j.cpc.2010.06.001

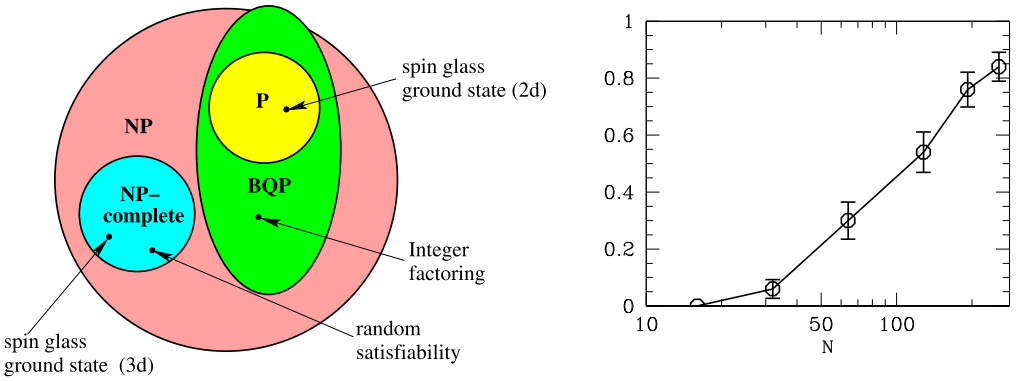

Fig. 1. Left: The probable relationship between complexity classes P, NP, NP-complete, and BQP (see text). The issue addressed in this paper is whether the quantum complexity class BQP might include NP-complete. Right: Fraction of instances with a first order transition, as a function of size for the Exact Cover problem.

order transition, and hence the running time will be exponentially long. We are currently modifying the form of the Hamiltonian used to represent the Exact Cover problem and studying other models, to see it the first order transition found here is special to Exact Cover or occurs more generally. Lack of space prevents us from discussing *related* work on first order phase transitions [5–7] in quantum versions of constraint satisfaction problems.

Acknowledgements

The work of A.P.Y. is supported by the National Security Agency (NSA) under Army Research Office (ARO) contract number W911NF-09-1-0391. The work of S.K. and V.N.S. is supported by National Security Agency's Laboratory of Physics Sciences and the NASA Ames NAS Supercomputing Center.

References

[1] P.W. Shor, Algorithms for quantum computing: discrete logarithms and factoring, in: S. Goldwasser (Ed.), Proc. 35th Symp. on Foundations of Computer Science, 1994, p. 124.
[2] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, D. Preda, A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem, Science 292 (2001) 472.
[3] A.P. Young, S. Knysh, V. Smelyanskiy, Size dependence of the minimum excitation gap in the Quantum Adiabatic Algorithm, Phys. Rev. Lett. 101 (2008) 170503.
[4] A.P. Young, S. Knysh, V. Smelyanskiy, First order phase transition in the Quantum Adiabatic Algorithm, Phys. Rev. Lett. 104 (2010) 020502.
[5] T. Jörg, F. Krzakala, G. Semerjian, F. Zamponi, First-order transitions for random optimization problems in a transverse field, arXiv:0911.3438, 2009.
[6] B. Altshuler, H. Krovi, J. Roland, Adiabatic quantum optimization fails for random instances of NP-complete problems, arXiv:0908.2782, 2009.
[7] E. Farhi, J. Goldstone, D. Gosset, S. Gutmann, H.B. Meyer, P. Shor, Quantum adiabatic algorithms, small gaps, and different paths, arXiv:0909.4766, 2009.